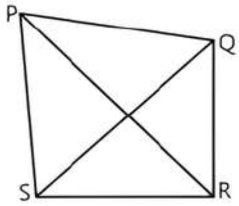
8 Quadrilaterals

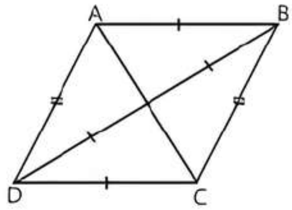
Fastrack Revision

- **Quadrilateral:** A closed figure formed by four line segments (with no three points collinear). In quadrilateral PQRS; PQ, QR, RS and SP are sides; ∠P, ∠Q, ∠R and ∠S are the four angles; PR and QS are diagonals; Pairs (PQ, QR), (QR, RS), (RS, SP) and (SP, PQ) are adjacent sides; Pairs (∠Q, ∠S) and (∠P, ∠R) are of opposite angles.



The sum of the four angles of a quadrilateral is 360°.

- **Parallelogram:** A quadrilateral in which both opposite pairs of sides are parallel, is said to be parallelogram. In figure, ABCD is a parallelogram with AB || CD and BC || AD.



- **Properties of a Parallelogram**
  - (i) Opposite sides are equal and parallel.
  - (ii) Opposite angles are equal.
  - (iii) Diagonals bisect each other.
  - (iv) Diagonals divided it into two congruent triangles.
  - (v) The sum of the adjacent angles of a parallelogram is 180°.
  - (vi) If a pair of opposite sides is parallel and equal, then it is a parallelogram.
- **Mid-point Theorem:** The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and is equal to half of it.

  Converse of Mid-point Theorem: The line drawn through the mid-point of one side of a triangle, parallel to another side bisects the third side.

Knowledge BOOSTER

1. *A trapezium is not a parallelogram (as only one pair of opposite sides is parallel in a trapezium and we require both pairs to be parallel in a parallelogram).*
2. *The figure formed by joining the mid-points of the adjacent sides of a quadrilateral (or rhombus/ square) is a parallelogram (or rectangle/square).*

Practice Exercise

Multiple Choice Questions

Q 1. The consecutive angles of a parallelogram are:

a. complementary
b. supplementary
c. equal
d. None of the above

Q 2. If in a parallelogram its diagonals bisect each other and are equal, then it is a:

a. square　b. rectangle
c. rhombus　d. parallelogram

Q 3. The figure formed by joining the mid-points of the adjacent sides of a rhombus is a:

a. rhombus　b. square
c. rectangle　d. parallelogram

Q 4. Which of the following is not true for a parallelogram?

a. Diagonals bisect each other
b. Opposite sides are equal
c. Opposite angles are equal
d. Opposite angles are bisected by the diagonals

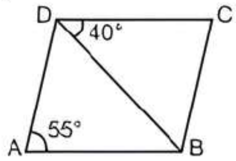
Q 5. If one angle of a parallelogram is 24 less than twice the smallest angle, then the largest angle of the parallelogram is:

a. 68°　b. 102°　c. 112°　d. 136°

Q 6. If ABCD is a parallelogram with two adjacent angles ∠A = ∠B, then the parallelogram is a:

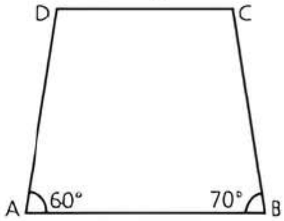
a. rectangle　b. rhombus
c. trapezium　d. kite

Q 7. In the given figure, ABCD is a parallelogram in which $\angle BDC = 40°$ and $\angle BAD = 55°$, then $\angle CBD$ is equal to:



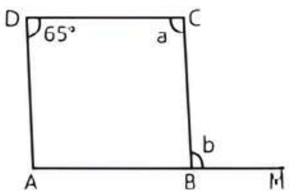
a. 80° b. 70° c. 90° d. 85°

Q 8. In the given figure AB || CD, then measure $\angle C$ is:



a. 120° b. 110° c. 115° d. 118°

Q 9. If ABCD is a parallelogram in which $\angle ADC = 65°$ and AB is produced to point M as shown in the figure. Then, $a + b$ is:



a. 235° b. 230° c. 225° d. 0°

Q 10. Diagonals of quadrilateral ABCD bisect each other. If $\angle A = 45°$, then the value of $\angle B$ is:

a. 90° b. 45°
c. 135° d. 120°

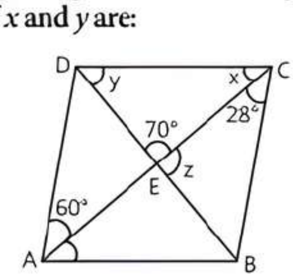
Q 11. If angles A, B, C and D of a quadrilateral ABCD, taken in order are in the ratio 3 : 7 : 6 : 4, then ABCD is a:

a. rhombus b. parallelogram
c. trapezium d. kite

Q 12. The diagonals AC and BD of a parallelogram ABCD intersect each other at the point O. If $\angle ACB = 32°$ and $\angle AOB = 70°$, then $\angle DBC$ is equal to:

a. 24° b. 86° c. 38° d. 32°

Q 13. In the given figure, ABCD is a parallelogram, the values of x and y are:



a. 30°, 75° b. 32°, 78°
c. 36°, 74° d. 35°, 70°

Q 14. In ΔABC, AB = 6 cm, BC = 9 cm and AC = 8 cm. If D and E are respectively the mid-point of AB and BC, then the length of DE is:

a. 4 cm b. 6 cm c. 5 cm d. 4.5 cm

Assertion & Reason Type Questions

Directions (Q.Nos. 15-18): *In the following questions, a statement of Assertion (A) is followed by a statement of a Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
c. Assertion (A) is true but Reason (R) is false.
d. Assertion (A) is false but Reason (R) is true.

Q 15. Assertion (A): The opposite angles of a parallelogram are $(2x - 2)°$ and $(52 - x)°$. The measure of one of the angle is 34°.

Reason (R): Opposite angles of a parallelogram are equal.

Q 16. Assertion (A): In ΔABC, median AD is produced to E, such that AD = DE. Then, ABEC is a parallelogram.

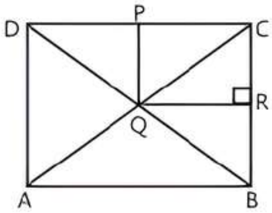
Reason (R): Diagonals AE and BC bisect each other at right angles.

Q 17. Assertion (A): Diagonals AC and BD of a parallelogram ABCD intersect each other at point O. If $\angle BCA = 35°$ and $\angle AOB = 65°$, then $\angle DBC = 30°$.

Reason (R): The adjacent angles of a parallelogram is supplementary.

Q 18. Assertion (A): ABCD and PQRC are rectangles and Q is a mid point of AC. Then DP = PC.

Reason (R): The line segment joining the mid-point of any two sides of a triangle is parallel to the third side and equal to half of it.



Fill in the Blanks Type Questions

Q 19. The .... angles of a parallelogram are equal.

Q 20. The diagonals of a rectangle are .... and .... each other.

Q 21. The quadrilateral formed by joining the mid-points of the consecutive sides of a square is a ....

Q 22. The line segment joining the mid-points of the two sides of the triangle is .... to the third side.

True/False Type Questions

Q 23. All the angles of the quadrilateral are obtuse.

Q 24. Out of four points A, B, C, D in plane, three of them are collinear. Then, a quadrilateral can be formed from these points.

Solutions

1. (b) supplementary
2. (b) rectangle
3. (c) rectangle
4. (d) opposite angles are bisected by the diagonals
5. (c) Let θ be the smallest angle of a parallelogram, then the other larger angle will be $2\theta - 24°$.

TiP

Opposite angles of a parallelogram are equal.

The sum of all angles of a parallelogram is 360°.

$\therefore \theta + 2\theta - 24° + \theta + 2\theta - 24° = 360°$

$\Rightarrow 6\theta - 48° = 360°$

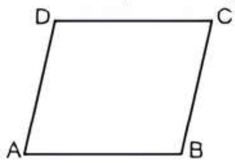
$\Rightarrow \theta = \frac{408°}{6} = 68°$

$\therefore$ The largest angle is $2\theta - 24°$

$= 2 \times 68° - 24°$

$= 136° - 24° = 112°$

6. (a) Given, ABCD is a parallelogram.



Therefore $\angle A + \angle B = 180°$

[Sum of adjacent angles of a parallelogram is 180°]

$\Rightarrow \angle A + \angle A = 180°$ [$\because \angle A = \angle B$ given]

$\Rightarrow \angle A = 90°$

$\Rightarrow \angle B = 90°$

Hence, ABCD is a rectangle.

7. (d) Given, $\angle BDC = 40°$

TiP

Opposite angles of a parallelogram are equal.

Here, $\angle BCD = \angle BAD = 55°$

In ΔBCD, use angle sum property of a triangle,

$\angle BDC + \angle BCD + \angle CBD = 180°$

$\Rightarrow 40° + 55° + \angle CBD = 180°$

$\Rightarrow \angle CBD = 85°$

8. (b) Given AB || CD, therefore the sum of two adjacent angles is 180°.

$\Rightarrow \angle B + \angle C = 180°$

$\Rightarrow 70° + \angle C = 180°$

$\Rightarrow \angle C = 110°$

9. (b) Given ABCD is a parallelogram, therefore sum of adjacent angles is 180°.

i.e. $\angle D + \angle C = 180°$

$65° + a = 180°$

$\Rightarrow a = 115°$

And opposite angles of a parallelogram are equal.

$\therefore \angle B = \angle D$

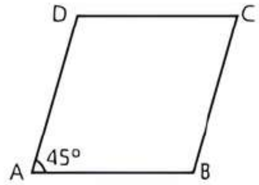
$\angle B = 65°$

$\therefore \angle CBM = 180° - \angle B$

$= 180° - 65° = 115°$

$\therefore a + b = 115° + 115° = 230°$

10. (c) Given, diagonals of a quadrilateral bisect each other, so it is a parallelogram.



Therefore, the sum of co-interior angles is 180°.

$\Rightarrow \angle A + \angle B = 180°$

$\Rightarrow 45° + \angle B = 180° \Rightarrow \angle B = 135°$

11. (c) Let angles of a quadrilateral be $A = 3x$, $B = 7x$, $C = 6x$ and $D = 4x$.

Then, sum of all angles of a quadrilateral be 360°.

$\therefore A + B + C + D = 360°$

$\Rightarrow 3x + 7x + 6x + 4x = 360°$

$\Rightarrow 20x = 360° \Rightarrow x = 18°$

$\therefore A = 3 \times 18 = 54°$ $\quad B = 7 \times 18 = 126°$

$C = 6 \times 18 = 108°$ and $D = 4 \times 18 = 72°$

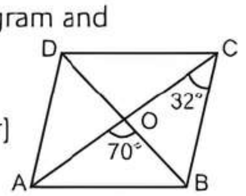
Here, we see that, neither pair of angles are equal nor sum of adjacent angles is 180°.

But, here $\angle A + \angle B = 54° + 126° = 180°$

and $\angle C + \angle D = 108° + 72° = 180°$

So, ABCD is a trapezium.

12. (c) Given, ABCD is a parallelogram and $\angle ACB = 32°$, $\angle AOB = 70°$.



Also, $\angle AOB + \angle BOC = 180°$ [Linear pair]

$\Rightarrow 70° + \angle BOC = 180°$

$\Rightarrow \angle BOC = 110°$

In ΔBOC, use angle sum property of a triangle.

$\angle BOC + \angle OCB + \angle OBC = 180°$

$\Rightarrow 110° + 32° + \angle OBC = 180°$ [$\because \angle OCB = \angle ACB = 32°$]

$\Rightarrow \angle OBC = 38°$

$\Rightarrow \angle DBC = 38°$ [$\because \angle OBC = \angle DBC = 38°$]

13. (b) Given, ABCD is a parallelogram.

Therefore, $\angle DCB = \angle DAB$

[Opposite angles of a parallelogram are equal]

$\therefore \quad x + 28° = 60°$

$\Rightarrow \quad x = 32°$

In ΔCDE, using angle sum property of a triangle,

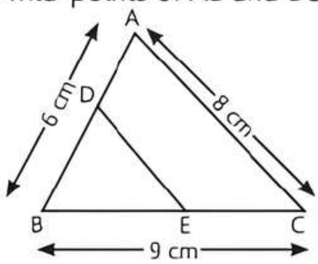
$\angle CDE + \angle DEC + \angle DCE = 180°$

$\Rightarrow \quad y + 70° + 32° = 180°$

$\Rightarrow \quad y = 78°$

14. (a) In ΔABC, we have

AB = 6 cm, BC = 9 cm and AC = 8 cm. Since D and E are the mid-points of AB and BC respectively.



By mid-point theorem,

DE || AC

and $DE = \frac{1}{2}AC = \frac{8}{2}$

$= 4$ cm

15. (a) **Assertion (A):** Given opposite angles of a parallelogram are equal.

$\therefore \quad (2x - 2)° = (52 - x)°$

$\Rightarrow \quad 2x + x = 52° + 2° \quad \Rightarrow \quad 3x = 54°$

$\Rightarrow \quad x = 18°$

Then the angles of a parallelogram are

$(2x - 2)° = (2 \times 18 - 2)° = 34°$

and $(52 - x)° = (52 - 18)° = 34°$

Hence, one of the angle of a parallelogram is 34°

So, Assertion (A) is true.

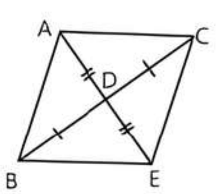
Reason (R): It is also true that opposite angles of a parallelogram are equal.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

16. (c) **Assertion (A):** Given in ΔABC, AD is median such that

AD = DE

Also, BD = DC.



It means in quadrilateral ABEC, diagonals AE and BC bisect each other at point D.

Therefore, ABEC is a parallelogram.

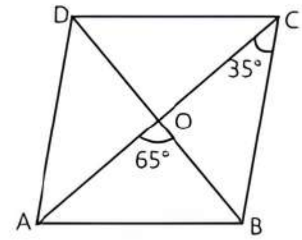
So, Assertion (A) is true.

Reason (R): In given figure diagonals are not right angled.

So, Reason (R) is false.

Hence, Assertion (A) is true but Reason (R) is false.

17. (b) **Assertion (A):** Given, ABCD is a parallelogram.



$\Rightarrow \quad \angle BCA = 35°$

or $\quad \angle BCO = 35°$

$\angle BOA + \angle BOC = 180°$ [Linear pair]

$\Rightarrow \quad \angle BOC = 180° - 65° = 115°$

In Δ BOC, use angle sum property of a triangle.

$\angle OBC + \angle BOC + \angle BCO = 180°$

$\Rightarrow \angle DBC + 115° + 35° = 180°$ [$\because \angle OBC = \angle DBC$]

$\Rightarrow \quad \angle DBC = 30°$

So, Assertion (A) is true.

Reason (R): It is true to say that adjacent angles of a parallelogram is supplementary.

Hence, both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).

18. (b) **Assertion (A):** In right angled ΔADC, Q is the mid-point of AC such that PQ || AD.

Therefore, P is the mid-point of DC.

[By converse of mid-point theorem]

DP = PC

So, Assertion (A) is true.

Reason (R): It is also true to say that the line segment joining the mid-point of any two sides of a triangle is parallel to the third side and equal to half of it.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

19. opposite **20.** equal, bisect

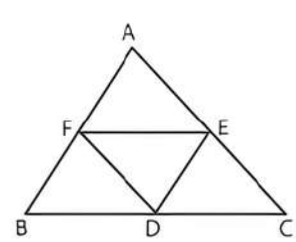
21. square **22.** parallel

23. False **24.** False

Case Study Based Questions

Case Study 1

A metal marker has a triangular shaped metal. He welded another triangle on the mid-points of that metal, such that it appears like the following figure:



In the above figure, D, E and F are the mid-points of BC, AC and AB.

On the basis of the above information, solve the following questions:

Q 1. DE is equal to:

a. AF b. $\frac{1}{2}$AB c. BF d. All of these

Q 2. If FE = FD, then which of the following relation is correct:

a. AC = AB b. ∠ FED = ∠ ECD
c. BC = AC d. ∠ CAB = ∠ AFD

Q 3. Which type of quadrilateral BDEF?

a. Parallelogram b. Square
c. rectangle d. Trapezium

Q 4. Identify the correct relation:

a. $FD = \frac{1}{2} AB$ b. AE + FD = AC
c. AB – DE = AC d. None of these

Q 5. The sum of adjacent angles in a parallelogram is:

a. 90° b. 145°
c. 180° d. None of these

Solutions

1. (d) ∵ D and E are the mid-points of side BC and AC.

∴ By mid-point theorem, $DE = \frac{1}{2} AB$

But $\frac{1}{2} AB = AF = BF$

∴ $DE = \frac{1}{2} AB = AF = BF$

So, option (d) is correct.

2. (c) $FD = \frac{1}{2} AC$ [By mid-point theorem]

and $FE = \frac{1}{2} BC$

Given, FD = FE

∴ $\frac{1}{2} AC = \frac{1}{2} BC$

AC = BC

So, option (c) is correct.

3. (a) In quadrilateral BDEF; FE = BD and DE = BF, so quadrilateral is a parallelogram.
So, option (a) is correct.

4. (b) ∵ $FD = \frac{1}{2} AC$ [By mid-point theorem] ...(1)

∵ E is the mid-point of AC.

∴ $AE = \frac{1}{2} AC$ ...(2)

Adding eqs. (1) and (2), we get

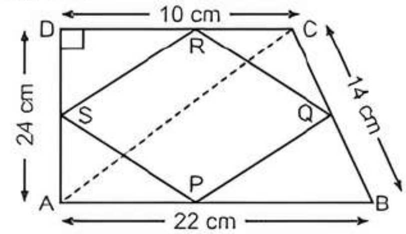
AE + FD = AC,

So, option (b) is correct.

5. (c) The sum of pair of adjacent angles in a parallelogram is 180°.
So, option (c) is correct.

Case Study 2

Person A has a quadrilateral shaped paper which he cut from a circular paper. Person B joined the mid-points of all sides and another quadrilateral was formed.



Above figure shows how the paper appears, side AB = 22 cm, BC = 14 cm, CD = 10 cm and AD = 24 cm.

On the basis of the above information, solve the following questions:

Q 1. The measure of diagonal AC is:

a. 13 cm b. 30 cm c. 28 cm d. 26 cm

Q 2. If PQ || AC, then the measure of PQ is:

a. 15 cm b. 13 cm
c. 17 cm d. 19 cm

Q 3. Quadrilateral PQRS is which of type quadrilateral?

a. Rhombus b. Rectangle
c. Parallelogram d. Trapezium

Q 4. While proving quadrilateral is a rectangle, choose the correct option:

a. by showing opposite sides equal and each adjacent angle is 90°
b. by proving diagonals are equal
c. by proving all angles 90°
d. by proving all of the above

Q 5. In any quadrilateral, the sum of all angles is:

a. 250° b. 360° c. 290° d. 270°

Solutions

1. (d) ∵ Δ ADC is a right angled triangle.

∴ $AC^2 = AD^2 + DC^2$ [Use Pythagoras theorem]

$AC^2 = 24^2 + 10^2$

$AC^2 = 576 + 100$

$AC^2 = 676$

$AC = \sqrt{676}$

AC = 26 cm

So, option (d) is correct.

2. (b) In ΔABC, P and Q are the mid-points of side AB and BC.

∴ $PQ = \frac{1}{2} AC$ [By mid-point theorem]

$PQ = \frac{1}{2} \times 26$

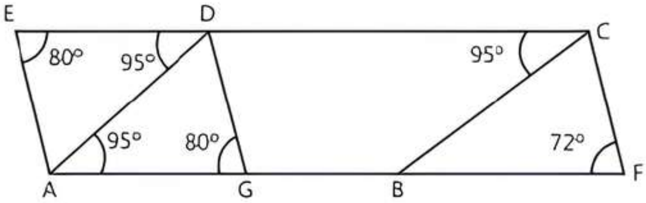
PQ = 13 cm

So, option (b) is correct.

3. (c) Since, P, Q, R and S are the mid-point of the sides AB, BC, CD and DA. Therefore, joining adjacent mid-point forms a parallelogram.
So, option (c) is correct.
4. (d) by proving all of the above
So, option (d) is correct.
5. (b) The sum of all angles in a quadrilateral is 360°.
So, option (b) is correct.

Case Study 3

A parallelogram shape park ABCD is in the middle of the city. Municipality decided to increase its area, so at the left side of park a triangle AED was added and on the right side triangle BFC was added. At point G on AB, municipality put a swing.



On the basis of the above information, solve the following questions:

Q 1. Prove that AGDE is a parallelogram when ED || AG, AE || DG.

Q 2. Find the value of ∠ADC.

Q 3. Find the value of ∠ DCF.

Solutions

1. Given, AE || DG and ED || AG

$\angle EDA = \angle DAG = 95°$ and $\angle AGD = 80°$

In Δ AED and Δ DGA,

$\angle DGA = \angle AED$ [Each 80°]

$\angle EDA = \angle DAG$ [Each 95°]

$AD = DA$ [Common]

$\therefore \Delta AED \cong \Delta DGA$ (by AAS congruence rule)

$\therefore AE = DG$ [by CPCT]

$ED = AG$ [by CPCT]

If in a quadrilateral each opposite sides are equal, then the quadrilateral is a parallelogram.

So, AGDE is a parallelogram. **Hence proved**

2. In parallelogram ABCD, $\angle DAG + \angle ADC + \angle DCB + \angle ABC = 360°$ [Sum of angles of a parallelogram]

$95° + \angle ADC + 95° + \angle ABC = 360°$

$2\angle ADC + 190° = 360°$ [$\because \angle ADC = \angle ABC$]

$2\angle ADC = 360° - 190°$

$2\angle ADC = 170°$

$\angle ADC = 85°$

3. In quadrilateral AFCD,

$\angle DAG + \angle ADC + \angle AFC + \angle DCF = 360°$

[Angle sum property of a quadrilateral]

$95° + 85° + 72° + \angle DCF = 360°$

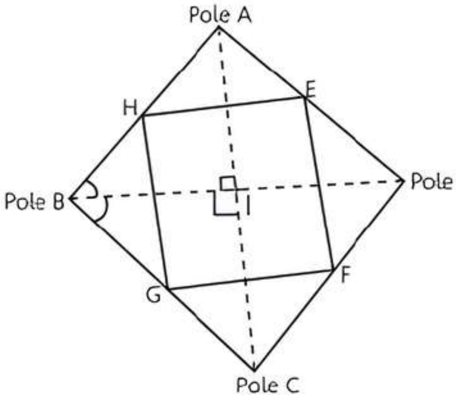
$\angle DCF = 360° - 95° - 85° - 72°$

$\angle DCF = 108°$

Case Study 4

Due to frequent robberies in the colony during night. The secretary with the members together decides to attach more lights besides the street light set by municipality. There are poles on which lights are attached.

These 4 poles are connected to each other through wire and they form a quadrilateral. Light from pole B focus light on mid-point G of wire between pole C and B, from pole C focus light on mid-point F of wire between pole C and pole D. Similarly pole D and pole A focus light on the mid-point E and H respectively.



On the basis of the above information, solve the following questions:

Q 1. If BD is the bisector of ∠B then prove that I is the mid-point of AC.

Q 2. Prove that quadrilateral EFGH is a parallelogram.

Q 3. Is it true that every parallelogram is a rectangle?

Solutions

1. In ΔBIA and ΔBIC,

$\angle ABI = \angle CBI$ [$\because$ BD is the bisector of $\angle B$]

$BI = BI$ [Common]

$\angle BIA = \angle BIC$ [Each 90°]

$\therefore$ $\Delta BIA \cong \Delta BIC$ [SAS congruence rule]

$\therefore$ $AI = CI$ [CPCT]

It means I is the mid-point of AC. **Hence proved**

2. Here, $HG = \frac{1}{2}AC$ [By mid-point theorem]

and $EF = \frac{1}{2}AC$ [By mid-point theorem]

$GH \parallel EF$ and $HG = EF$

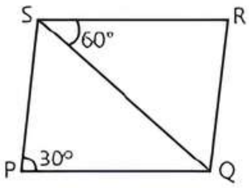
If in a quadrilateral opposite side is parallel and equal then the quadrilateral is a parallelogram. So, quadrilateral EFGH is a parallelogram.

3. It is false, because every parallelogram is not a rectangle.

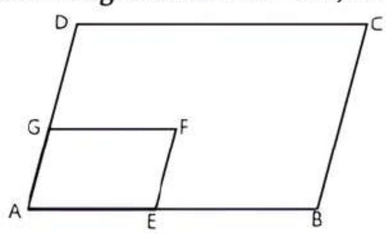
Very Short Answer Type Questions

Q 1. Two consecutive angles of a parallelogram are in the ratio 1 : 3, then what will be the smaller angle?

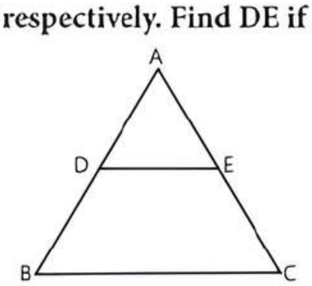
Q 2. In the given figure, PQRS is a parallelogram in which $\angle QSR = 60°$ and $\angle QPS = 30°$. Find $\angle RQS$.



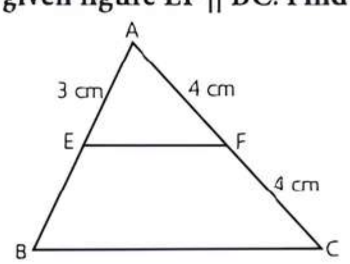
Q 3. In the following figure, ABCD and AEFG are two parallelograms. If $\angle C = 55°$, determine $\angle F$.



Q 4. In ΔABC, D and E are the mid-points of AB and AC respectively. Find DE if BC = 12 cm.



Q 5. In the given figure $EF \parallel BC$. Find BE.



Q 6. D, E, F are the mid-points of sides BC, CA and AB of ΔABC. If perimeter of ΔABC is 12.8 cm, then what is the perimeter of ΔDEF?

Short Answer Type-I Questions

Q 1. Two opposite angles of a parallelogram are $(3x - 2)°$ and $(63 - 2x)°$. Find all the angles of a parallelogram.

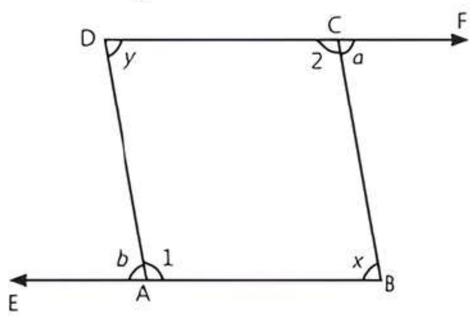
Q 2. In quadrilateral ABCD, AO and BO are the bisectors of $\angle A$ and $\angle B$ respectively, $\angle C = 60°$ and $\angle D = 40°$. Find $\angle AOB$.

Q 3. In a parallelogram, show that the angle bisectors of two adjacent angles intersect at a right angle.

Q 4. The angle between the two altitudes of a parallelogram through the vertex of an obtuse angle is 50°. Find the angles of a parallelogram.

Q 5. A diagonal of a rectangle is inclined to the one side of the rectangle at 35°. Find the acute angle between the diagonals.

Q 6. The sides BA and DC of a quadrilateral ABCD are produced as shown in given figure. Prove that $x + y = a + b$.

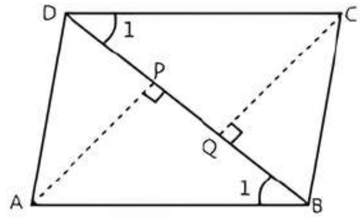


Q 7. ABCD is a rhombus such that $\angle ACB = 55°$, then find $\angle ADB$.

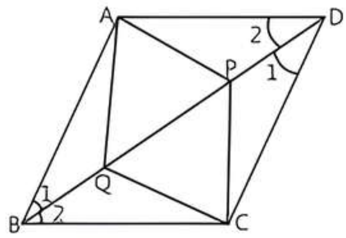
Short Answer Type-II Questions

Q 1. ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see figure). Show that:

(i) $\Delta APB \cong \Delta CQD$ (ii) $AP = CQ$



Q 2. In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see figure). Show that:

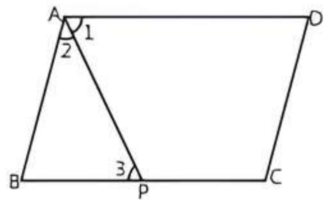


(i) $\Delta APD \cong \Delta CQB$

(ii) $AP = CQ$

(iii) $\Delta AQB \cong \Delta CPD$

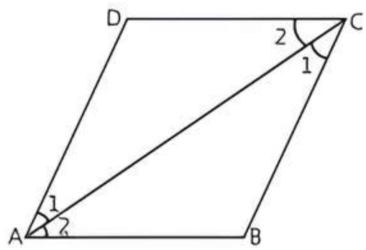
Q 3. P is the mid-point of side BC of parallelogram ABCD, such that $\angle 1 = \angle 2$ and $\angle 1 = \angle 3$. Prove that $AD = 2CD$.



Q 4. Diagonal AC of a parallelogram ABCD bisects $\angle A$ (see figure). Show that:

(i) AC bisects $\angle C$

(ii) ABCD is a rhombus.



Q 5. ABC is a triangle right-angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D such that $\angle ADM = \angle ACB$. Show that:

(i) D is the mid-point of AC.

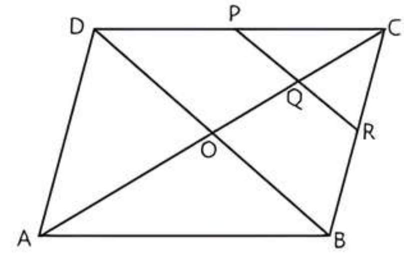
(ii) $MD \perp AC$.

(iii) $CM = MA = \frac{1}{2}AB$.

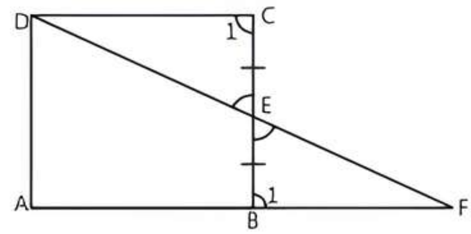
Q 6. D, E and F are the mid-points of the sides BC, CA and AB, respectively of an equilateral triangle ABC. Show that ΔDEF is also an equilateral triangle.

Q 7. ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

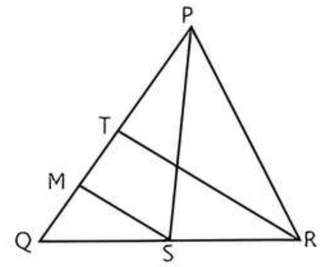
Q 8. In the following figure, ABCD is a parallelogram in which P is the mid-point of DC and Q is a point on AC, such that $CQ = \frac{1}{4}AC$. Also, PQ when produced meets BC at R. Prove that R is the mid-point of BC.



Q 9. In the figure, ABCD is a parallelogram and E is the mid-point of side BC. DE and AB are produced to meets at F. Prove that $AF = 2AB$.



Q 10. In the following figure, PS and RT are medians of ΔPQR and $SM \parallel RT$. Prove that $QM = \frac{1}{4}PQ$.



Long Answer Type Questions

Q 1. Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Q 2. In ΔABC and ΔDEF, $AB = DE$, $AB \parallel DE$, $BC = EF$ and $BC \parallel EF$. Vertices A, B and C are joined to vertices D, E and F respectively (see figure). Show that:

(i) Quadrilateral ABED is a parallelogram.

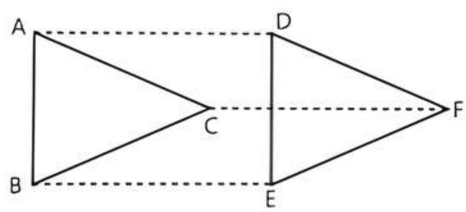
(ii) Quadrilateral BEFC is a parallelogram.

(iii) $AD \parallel CF$ and $AD = CF$.

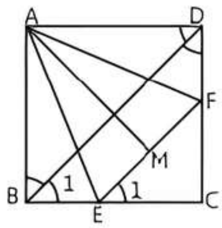
(iv) Quadrilateral ACFD is a parallelogram.

(v) $AC = DF$.

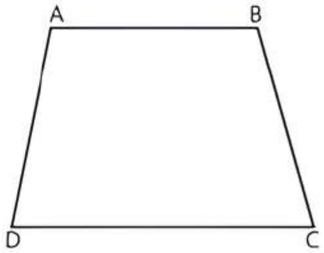
(vi) $\Delta ABC \cong \Delta DEF$.



Q 3. In the given figure, ABCD is a square and $EF \parallel BD$. M is the mid-point of EF. Prove that AM bisects $\angle BAD$.



Q 4. ABCD is a trapezium in which AB || CD and AD = BC (see figure).



Show that:

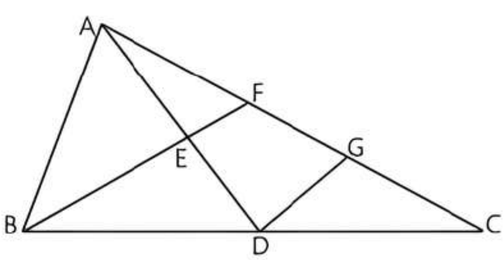
(i) $\angle A = \angle B$ (ii) $\angle C = \angle D$

(iii) $\Delta ABC \cong \Delta BAD$

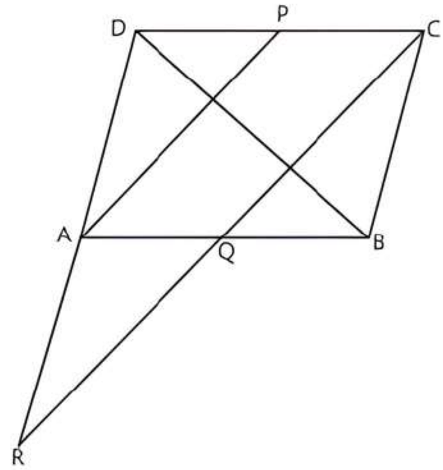
(iv) Diagonal AC = Diagonal BD

Q 5. Prove that the bisector of the angles of a parallelogram encloses a rectangle.

Q 6. In given figure, AD is the median of ΔABC. E is the mid-point of AD. DG||BF. Prove that AC = 3 AF.



Q 7. P is the mid-point of the side CD of a parallelogram ABCD. A line through C parallel to PA intersects AB at Q and DA produced to R. Prove that DA = AR and CQ = QR.



Q 8. Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Q 9. Two parallel lines l and m are intersected by a transversal p. Show that the quadrilateral formed by the bisectors of interior angles is a rectangle.

Q 10. E and F are respectively the mid-points of the non-parallel sides AD and BC of a trapezium ABCD.

Prove that EF || AB and $EF = \frac{1}{2}(AB + CD)$.

Q 11. ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Q 12. Prove that the line segment joining the mid-points of the diagonals of a trapezium is parallel to the parallel sides and equal to half of their difference.

Solutions

Very Short Answer Type Questions

1. Let the consecutive angles be x and $(3x)$.

$\therefore \quad x + (3x) = 180°$

$\Rightarrow \quad 4x = 180°$

$\Rightarrow \quad x = 45°$

Hence, smaller angle is 45°.

2. Given, $\angle QSR = 60°$ and $\angle QPS = 30°$

Since, opposite angles of a parallelogram are equal.

$\therefore \quad \angle QRS = \angle QPS = 30°$

In ΔQRS,

$\angle QRS + \angle QSR + \angle RQS = 180°$

[Use angle sum property of a triangle]

$\Rightarrow \quad 30° + 60° + \angle RQS = 180°$

$\Rightarrow \quad 90° + \angle RQS = 180°$

$\Rightarrow \quad \angle RQS = 90°$

3. Given, ABCD is a parallelogram.

$\therefore \quad \angle A = \angle C = 55°$ [Opposite angles of a parallelogram]

Also, AEFG is a parallelogram.

$\therefore \quad \angle F = \angle A = 55°$ [Opposite angles of a parallelogram]

4. Given, D is the mid-point of AB and E is the mid-point of AC.

By mid-point theorem,

$$DE = \frac{1}{2}BC = \frac{1}{2} \times 12 \text{ cm} = 6 \text{ cm}$$

5. Since AF = FC = 4 cm,

$\therefore$ F is the mid-point of AC.

Also, EF || BC

By the converse of mid-point theorem,

E is mid-point of AB.

Hence, BE = AE = 3 cm

6. Given, perimeter of $\Delta ABC = 12.8$ cm

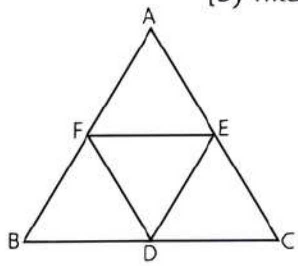
Perimeter of ΔABC

$= AB + BC + CA = 12.8$ cm

Perimeter of ΔDEF

$= DE + EF + FD$

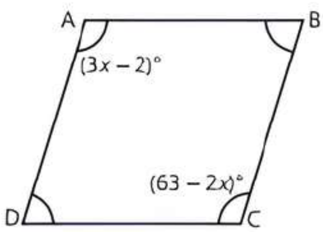
$= \frac{AB}{2} + \frac{BC}{2} + \frac{CA}{2}$ [By mid-point theorem]



$= \frac{1}{2}(AB + BC + CA) = \frac{12.8}{2} = 6.4$ cm

Short Answer Type-I Questions

1. Since, opposite angles of a parallelogram are equal.



$\therefore \quad (3x - 2)° = (63 - 2x)°$

$\Rightarrow \quad 5x = 65 \quad \Rightarrow \quad x = 13°$

So, $\angle A = (3x - 2)° = (3 \times 13 - 2)° = (39 - 2)° = 37°$

$\angle B = 180° - 37° = 143°$ $(\because \angle B = 180° - \angle A)$

$\angle C = (63 - 2x)° = (63 - 2 \times 13)°$

$= 63 - 26 = 37°$

and $\angle D = 180° - 37° = 143°$ $[\because \angle D = 180° - \angle C]$

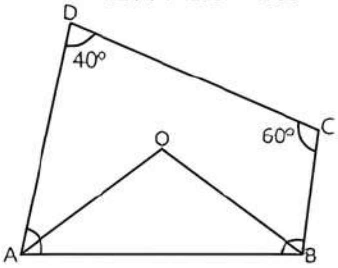
Hence, angles are 37°, 143°, 37° and 143°.

2. As we know that, sum of all angles of a quadrilateral is 360°.

$\therefore \quad \angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow \quad \angle A + \angle B + 60° + 40° = 360°$

$\Rightarrow \quad \angle A + \angle B = 260°$



Divide both sides by 2, we get

$$\frac{1}{2}\angle A + \frac{1}{2}\angle B = 130°$$

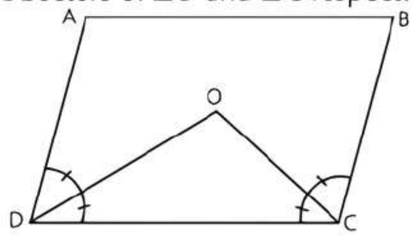
In Δ AOB,

$$\frac{\angle A}{2} + \frac{\angle B}{2} + \angle AOB = 180°$$

$\Rightarrow \quad 130° + \angle AOB = 180°$

$\Rightarrow \quad \angle AOB = 50°$

3. Let ABCD be a parallelogram and DO and CO are the bisectors of $\angle D$ and $\angle C$ respectively.



In parallelogram ABCD,

$\angle D + \angle C = 180°$

[Sum of two adjacent angles]

$\Rightarrow \quad \frac{1}{2}\angle D + \frac{1}{2}\angle C = \frac{1}{2} \times 180°$ [Dividing by 2]

$\Rightarrow \quad \angle ODC + \angle OCD = 90°$ ...(1)

[$\because$ DO and CO are the bisectors of $\angle D$ and $\angle C$ respectively]

In Δ DOC,

$\angle DOC + \angle ODC + \angle OCD = 180°$

[Angle sum property of a triangle]

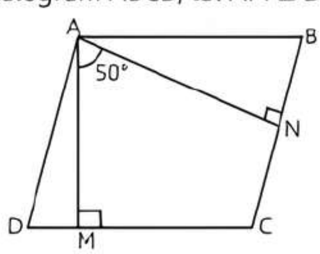
$\Rightarrow \quad \angle DOC + 90° = 180°$ [From eq. (1)]

$\Rightarrow \quad \angle DOC = 180° - 90°$

$\Rightarrow \quad \angle DOC = 90°$

So, the angle bisectors of two adjacent angles intersect at a right angle. **Hence proved**

4. In parallelogram ABCD, let AM $\perp$ DC and AN $\perp$ BC.



In quadrilateral AMCN,

$\angle A + \angle M + \angle C + \angle N = 360°$ [Angle sum property of a quadrilateral]

$\therefore \quad \angle A + \angle C = 360° - 180° = 180°$

$[\because \angle M + \angle N = 180°]$

$\Rightarrow \quad 50° + \angle C = 180°$

$\Rightarrow \quad \angle C = 130°$

In parallelogram ABCD, $\angle A = \angle C = 130°$

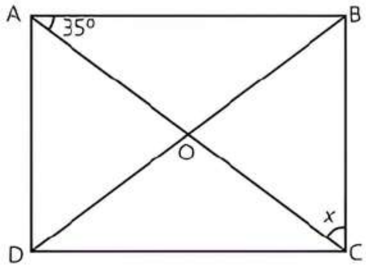
TiP

Adjacent angles of a parallelogram are supplementary.

$\therefore \quad \angle B = \angle D = 180° - 130°$

$= 50°$

5. Given, ABCD is a rectangle and diagonals of a rectangle bisect each other.



$\therefore$ AC = BD

$\Rightarrow \frac{1}{2}AC = \frac{1}{2}BD$

$\Rightarrow$ OA = OB

$\Rightarrow \angle OBA = \angle OAB$

[Angles opposite to equal sides are equal]

$\Rightarrow \angle OBA = 35°$ [$\because \angle OAB = 35°$]

In ΔAOB, $\angle AOB + \angle OAB + \angle OBA = 180°$

[Angle sum property of a triangle]

$\Rightarrow \angle AOB + 35° + 35° = 180°$

$\Rightarrow \angle AOB = 180° - 70° = 110°$

Therefore, $\angle AOB$ is an obtuse angle, but we have to find out acute angle between diagonals.

So, $\angle AOD + \angle AOB = 180°$ [Linear pair]

$\Rightarrow \angle AOD + 110° = 180°$

$\Rightarrow \angle AOD = 180° - 110°$

$\Rightarrow \angle AOD = 70°$

COMMON ERR(!)R

Sometimes the students do not read the question carefully. In haste they do a mistake of finding an acute angle instead of finding an obtuse angle.

6. From figure,

$\angle 1 + b = 180°$ [Linear pair]

$\Rightarrow \angle 1 = 180° - b$ ...(1)

Again,

$\angle 2 + a = 180°$ [Linear pair]

$\Rightarrow \angle 2 = 180° - a$ ...(2)

In quadrilateral ABCD,

$\angle A + \angle B + \angle C + \angle D = 360°$

[Sum of all interior angles in a quadrilateral]

$\Rightarrow \angle 1 + x + \angle 2 + y = 360°$ [From eqs. (1) and (2)]

$\Rightarrow 180° - b + x + 180° - a + y = 360°$

$\Rightarrow x + y = 360° - 360° + a + b$

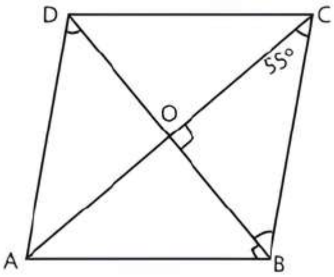
$\Rightarrow x + y = a + b$ **Hence proved**

7.

TiP

Diagonals of a rhombus bisect each other at right angle.

Here, $\angle BOC = 90°$



In right-angled ΔBOC,

$\angle BOC + \angle OCB + \angle OBC = 180°$

[Angle sum property of a triangle]

$\Rightarrow 90° + 55° + \angle OBC = 180°$

$\Rightarrow 145° + \angle OBC = 180°$

$\Rightarrow \angle OBC = 180° - 145°$

$\Rightarrow \angle OBC = 35°$

$\therefore \angle ADB = \angle OBC = 35°$

[Alternate interior angles as AD || BC]

COMMON ERR(!)R

Adequate practice is required of this type of question.

Short Answer Type-II Questions

1. **Given:** ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on BD.

To Prove: (i) $\Delta APB \cong \Delta CQD$

(ii) AP = CQ

Proof: (i) In right-angled ΔAPB and right-angled ΔCQD,

$\angle ABP = \angle CDQ$ [Given]

AB = DC [Opposite sides of a parallelogram]

$\angle APB = \angle CQD$ [Each 90°]

$\therefore \Delta APB \cong \Delta CQD$ [By ASA congruence rule]

(ii) $\because \Delta APB \cong \Delta CQD$ [Proved above]

$\therefore$ AP = CQ [By CPCT] **Hence proved**

2. **Given:** ABCD is a parallelogram and P and Q are points on diagonal BD such that DP = BQ.

To Prove: (i) $\Delta APD \cong \Delta CQB$

(ii) AP = CQ

(iii) $\Delta AQB \cong \Delta CPD$

Proof: (i) In ΔAPD and ΔCQB,

AD = BC [Opposite sides of a parallelogram]

$\angle ADP = \angle CBQ$ [Given]

DP = BQ [Given]

$\therefore \Delta APD \cong \Delta CQB$ [By SAS congruence rule]

(ii) $\because \Delta APD \cong \Delta CQB$ [Proved above]

$\therefore$ AP = CQ [By CPCT]

(iii) In ΔAQB and ΔCPD,

$AB = CD$

[Opposite sides of a parallelogram are equal]

$\angle ABQ = \angle CDP$ [Given]

$BQ = DP$ [Given]

$\therefore$ $\Delta AQB \cong \Delta CPD$ [By SAS congruence rule]

3. **Given:** P is the mid-point of side BC of parallelogram ABCD such that $\angle 1 = \angle 2$.

To Prove: AD = 2CD

Proof: In parallelogram ABCD,

$\angle 1 = \angle 3$ [Given]

But $\angle 1 = \angle 2$ [Given]

$\Rightarrow$ $\angle 2 = \angle 3$

$\Rightarrow$ $BP = BA$

[Sides opposite to equal angles are equal]

But P is the mid-point of BC.

So, $BP = \frac{1}{2}BC$

$\Rightarrow$ $AB = \frac{1}{2}AD$ [$\because$ BP = BA and BC = AD]

$\Rightarrow$ $CD = \frac{1}{2}AD$ [$\because$ AB = CD, opposite sides of a parallelogram]

$\Rightarrow$ $AD = 2CD$. **Hence proved**

4. **Given:** A parallelogram ABCD in which diagonal AC bisects $\angle A$, *i.e.*, $\angle DAC = \angle BAC$.

To Prove: (i) Diagonal AC bisects $\angle C$ *i.e.*, $\angle DCA = \angle BCA$.

(ii) ABCD is a rhombus.

Proof: (i) In parallelogram ABCD,

$\angle DAC = \angle BCA = \angle 1$ [Given]

$\angle BAC = \angle DCA = \angle 2$ [Given]

But $\angle DAC = \angle BAC$ [Given]

$\therefore$ $\angle BCA = \angle DCA$

So, AC bisects $\angle DCB$.

or AC bisects $\angle C$. **Hence proved**

(ii) In ΔABC and ΔADC,

$\angle BAC = \angle DAC$ [Given]

$AC = AC$ [Common]

and $\angle BCA = \angle DCA$ [Proved above]

$\therefore$ $\Delta ABC \cong \Delta ADC$

[By ASA congruence rule]

Thus, $BC = DC$ [By CPCT]

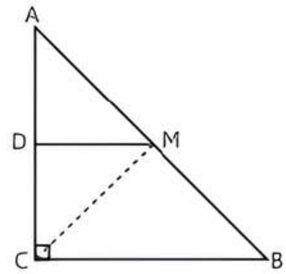
But $AB = DC$

[$\because$ Opposite sides of a parallelogram are equal]

$\therefore$ $AB = BC = DC = AD$

So, ABCD is a rhombus. **Hence proved**

5. **Given:** A triangle ABC, $\angle C = 90°$, M is the mid-point of AB and BC || DM.



To Prove: (i) D is the mid-point of AC.

(ii) $MD \perp AC$. (iii) $CM = MA = \frac{1}{2}AB$.

Construction: Join CM.

Proof: (i) In ΔABC,

M is the mid-point of AB. [Given]

BC || MD [Given]

$\therefore$ D is the mid-point of AC.

[By converse of mid-point theorem]

(ii) Now, $\angle ADM = \angle ACB$ [Given]

But $\angle ACB = 90°$ [Given]

$\therefore$ $\angle ADM = 90°$

But $\angle ADM + \angle CDM = 180°$ [Linear pair]

$\therefore$ $\angle CDM = 180° - 90° = 90°$

Hence, $MD \perp AC$

(iii) Again, $AD = DC$ ...(1)

[D is the mid-point of AC]

Now, in ΔADM and ΔCDM,

$AD = DC$ [From eq. (1)]

$\angle ADM = \angle CDM$ [Each 90°]

$DM = DM$ [Common]

$\therefore$ $\Delta ADM \cong \Delta CDM$ [By SAS congruence rule]

Thus, $MA = CM$ [By CPCT] ...(2)

Since M is mid-point of AB.

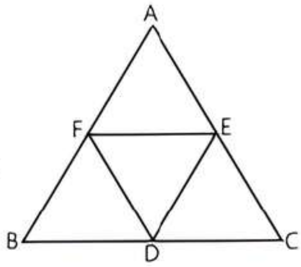
$\therefore$ $MA = \frac{1}{2}AB$ ...(3)

So, $CM = MA = \frac{1}{2}AB$ [From eqs. (2) and (3)]

Hence proved

6. **Given:** D, E and F are the mid-points of BC, CA and AB of Δ ABC.

To Prove: Δ DEF is an equilateral triangle.



Proof: In ΔABC, F is the mid-point of AB and E is the mid-point of AC.

So, by mid-point theorem,

$EF = \frac{1}{2}BC$ ...(1)

Similarly, $DE = \frac{1}{2}AB$ ...(2)

and $DF = \frac{1}{2}AC$ ...(3)

Since, ΔABC is an equilateral triangle.

$\therefore$ $AB = BC = AC$ [All sides are equal]

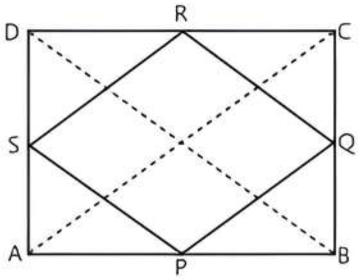
From eqs. (1), (2) and (3), we have

$EF = DE = DF$

So, ΔDEF is also an equilateral triangle.

Hence proved

7. **Given:** A rectangle ABCD in which P, Q, R, S are the mid-points of AB, BC, CD and DA respectively. PQ, QR, RS and SP are joined.



To Prove: PQRS is a rhombus.

Construction: Join AC and BD.

Proof: In ΔABC, P and Q are the mid-points of the sides AB and BC respectively.

$\therefore$ $PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ ...(1)

[By mid-point theorem]

Similarly, in ΔADC,

$SR \parallel AC$ and $SR = \frac{1}{2}AC$ ...(2)

From eqs. (1) and (2), we get

$PQ \parallel SR$ and $PQ = SR$ ...(3)

Now in quadrilateral PQRS, its one pair of opposite sides PQ and SR is parallel and equal. [From eq. (3)]

$\therefore$ PQRS is a parallelogram.

R is the mid-point of DC and Q is the mid-point of CB.

$\therefore$ $RQ \parallel BD$ and $RQ = \frac{1}{2}BD$ ...(4)

[By mid-point theorem]

S is the mid-point of AD and P is the mid-point of AB.

$\therefore$ $SP \parallel BD$ and $SP = \frac{1}{2}BD$ ...(5)

[By mid-point theorem]

From eqs. (4) and (5), we get

$RQ \parallel SP$ and $RQ = SP$ ...(6)

But $AC = BD$ [Diagonals of a rectangle]

$\therefore$ $PQ = QR$ [From eqs. (1) and (4)]

Thus, $PQ = QR = PS = SR$

So, PQRS is a rhombus. **Hence proved**

8. **Given:** ABCD is a parallelogram in which P is the mid-point of DC and $CQ = \frac{1}{4}AC$.

To Prove: R is the mid-point of BC.

Proof: Since diagonals of a parallelogram bisect each other.

$\therefore$ $OC = OA$

i.e., $OC = OA = \frac{1}{2}AC \Rightarrow AC = 2OC$

But $CQ = \frac{1}{4}AC$ [Given]

$= \frac{1}{4} \times 2OC = \frac{1}{2}OC$

Thus, Q is the mid-point of CO.

In ΔCDO,

P is the mid-point of CD [Given]

and Q is the mid-point of CO. [Proved above]

$\therefore$ $PQ \parallel DO$ [By mid-point theorem]

or $PR \parallel DB$

Now, in ΔCOB,

Q is the mid-point of OC. [Proved above]

$\therefore$ $QR \parallel OB$ [Proved above]

So, R is the mid-point of CB.

[By mid-point theorem]

Hence proved

9. **Given:** In parallelogram ABCD, E is the mid-point of side BC. Produce DE and AB meets at F.

To Prove: $AF = 2AB$.

Proof: In ΔDCE and ΔFBE,

$\angle DCE = \angle FBE = 1$ [Given]

$CE = BE$ [Given]

$\angle DEC = \angle BEF$ [Vertically opposite angles]

$\therefore$ $\Delta DCE \cong \Delta FBE$ [By ASA congruence rule]

Thus, $DC = FB$ [By CPCT]

Now, $AF = AB + BF$

$= AB + DC = AB + AB$ [$\because$ DC = AB]

$= 2AB$

$\therefore$ $AF = 2AB$ **Hence proved**

10. **Given:** PS and RT are the medians of ΔPQR, *i.e.,* S and T are the mid-points of QR and PQ respectively, *i.e.,* $SQ = SR = \frac{1}{2}QR$

and $PT = QT = \frac{1}{2}PQ$ and $SM \parallel RT$

To Prove: $QM = \frac{1}{4}PQ$.

Proof: In ΔQRT, S is the mid-point of QR and $SM \parallel RT$.

So, M is the mid-point of QT.

[By converse of mid-point theorem]

$\therefore$ $QM = MT = \frac{1}{2}QT$ ...(1)

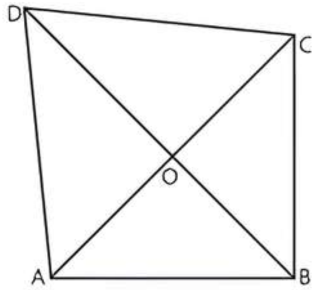
But $QT = \frac{1}{2}PQ$ [Given]

$QM = \frac{1}{2} \times \frac{1}{2}PQ$ [From eq. (1)]

$\Rightarrow$ $QM = \frac{1}{4}PQ$ **Hence proved**

Long Answer Type Questions

1. **Given:** A quadrilateral ABCD, in which diagonals AC and BD are equal and bisect each other at right angles.



To Prove: ABCD is a square.

Proof: Since ABCD is a quadrilateral whose diagonals bisect each other, so it is a parallelogram. Also, its diagonals bisect each other at right angles, therefore, ABCD is a rhombus.

$\Rightarrow$ AB = BC = CD = DA [Sides of a rhombus]

In ΔABC and ΔBAD,

AB = AB [Common]

BC = AD [Sides of a rhombus]

AC = BD [Given]

$\therefore$ $\Delta ABC \cong \Delta BAD$ [By SSS congruence rule]

Thus, $\angle ABC = \angle BAD$ [By CPCT]

But $\angle ABC + \angle BAD = 180°$

[Consecutive interior angles of rhombus]

$\therefore$ $\angle ABC = \angle BAD = 90°$

$\Rightarrow$ $\angle A = \angle B = \angle C = \angle D = 90°$

[Opposite angles of a rhombus]

$\Rightarrow$ ABCD is a rhombus whose angles are of 90° each.

So, ABCD is a square. **Hence proved**

2. **Given:** In ΔABC and ΔDEF, AB = DE, AB || DE, BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F.

To Prove: (i) ABED is a parallelogram.

(ii) BEFC is a parallelogram.

(iii) AD || CF and AD = CF

(iv) Quadrilateral ACFD is a parallelogram.

(v) AC = DF

(vi) $\Delta ABC \cong \Delta DEF$

Proof: (i) In quadrilateral ABED,

AB = DE and AB || DE. [Given]

$\Rightarrow$ ABED is a parallelogram.

[One pair of opposite sides is parallel and equal]

(ii) In quadrilateral BEFC,

BC = EF and BC || EF [Given]

$\Rightarrow$ BEFC is a parallelogram.

TiP

In parallelogram one pair of opposite sides is parallel and equal.

(iii) BE = CF and BE || CF [BEFC is a parallelogram]

AD = BE and AD || BE

[ABED is a parallelogram]

$\Rightarrow$ AD = CF and AD || CF

(iv) In quadrilateral ACFD, AD = CF and AD || CF

$\Rightarrow$ ACFD is a parallelogram.

[One pair of opposite sides is parallel and equal]

(v) In parallelogram AFCD, AC = DF

[Opposite sides of parallelogram are equal]

(vi) In ΔABC and ΔDEF,

AB = DE [Given]

BC = EF [Given]

AC = DF [Proved above]

$\therefore$ $\Delta ABC \cong \Delta DEF$ [By SSS congruence rule]

Hence proved

3. Given, ABCD is a square and BD is a diagonal.

$\therefore$ $\angle CBD = \angle CDB = \frac{1}{2} \times 90° = 45°$

[$\because$ Diagonal of a square bisect each angle at the vertex]

Also, given $\angle CEF = \angle CBD = 45°$

and $\angle CFE = \angle CDB = 45°$

[$\because$ ΔBCD is a right angled triangle

$\therefore$ $\angle CDB = \angle CBD = 45°$]

$\Rightarrow$ CE = CF

[$\because$ Sides opposite to equal angles are equal]

$\Rightarrow$ BC − CE = CD − CF [$\because$ BC = CD]

BE = DF ...(1)

Now, in Δ ABE and Δ ADF,

AB = AD [Sides of a square]

BE = DF [From eq. (1)]

$\angle ABE = \angle ADF$ [Each 90°]

So, $\Delta ABE \cong \Delta ADF$ [By SAS congruence rule]

Then, AE = AF [By CPCT] ...(2)

and $\angle BAE = \angle DAF$ ...(3)

Now, in Δ AEM and Δ AFM,

AE = AF [From eq. (2)]

ME = MF [M is mid-point of EF]

AM = AM [Common side]

$\therefore$ $\Delta AEM \cong \Delta AFM$ [By SSS congruence rule]

So, $\angle EAM = \angle FAM$ [By CPCT] ...(4)

On adding eqs. (3) and (4), we get

$\angle BAE + \angle EAM = \angle DAF + \angle FAM$

$\Rightarrow$ $\angle BAM = \angle DAM$

i.e., AM bisects $\angle BAD$. **Hence proved**

4. **Given:** In trapezium ABCD, AB || DC and AD = BC.

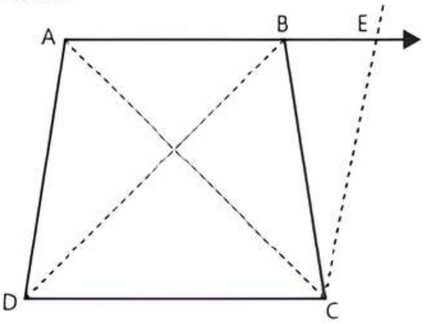
To Prove: (i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\Delta ABC \cong \Delta BAD$

(iv) Diagonal AC = Diagonal BD

Construction: Join AC and BD. Extend AB and draw a line through C parallel to DA meeting AB produced at E.



Proof: (i) Since AB || DC

$\Rightarrow$ AE || DC ...(1)

and AD || CE [By construction] ...(2)

$\Rightarrow$ ADCE is a parallelogram.

[Opposite pairs of sides are parallel]

$\angle A + \angle E = 180°$ ...(3)

[Co-interior angles]

$\angle B + \angle CBE = 180°$ [Linear pair] ...(4)

AD = EC ...(5)

[Opposite sides of a parallelogram are equal]

AD = BC [Given] ...(6)

$\Rightarrow$ BC = EC [From eqs. (5) and (6)]

$\Rightarrow$ $\angle E = \angle CBE$ ...(7)

[Angles opposite to equal sides are equal]

$\therefore$ $\angle B + \angle E = 180°$ ...(8)

[From eqs. (4) and (7)]

Now, from eqs. (3) and (8),

$\angle A + \angle E = \angle B + \angle E$

$\Rightarrow$ $\angle A = \angle B$

(ii) $\angle A + \angle D = 180°$ [Consecutive

$\angle B + \angle C = 180°$ interior angles]

$\Rightarrow$ $\angle A + \angle D = \angle B + \angle C$

$\Rightarrow$ $\angle D = \angle C$ [$\because \angle A = \angle B$]

or $\angle C = \angle D$

(iii) In Δ ABC and ΔBAD,

AD = BC [Given]

$\angle A = \angle B$ [Proved above]

AB = AB [Common]

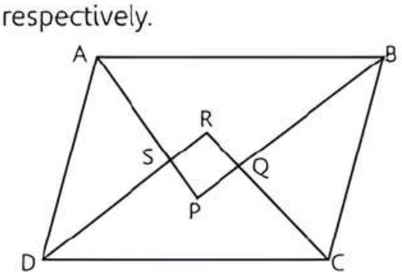
$\therefore$ $\Delta ABC \cong \Delta BAD$ [By SAS congruence rule]

(iv) $\because$ $\Delta ABC \cong \Delta BAD$

$\therefore$ Diagonal AC = Diagonal BD [By CPCT]

Hence proved

5. **Given:** ABCD is a parallelogram in which, AP, BP, CR and DR are the bisectors of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ respectively.



To Prove: PQRS is a rectangle.

Proof: In parallelogram ABCD,

$\angle A + \angle D = 180°$ [Sum of co-interior angles as, AB || DC]

$\Rightarrow$ $\frac{1}{2}\angle A + \frac{1}{2}\angle D = \frac{180°}{2}$ [Dividing by 2]

$\Rightarrow$ $\angle DAS + \angle ADS = 90°$

In Δ ASD,

$\angle ASD + \angle DAS + \angle ADS = 180°$

[Angle sum property of a triangle]

$\Rightarrow \angle ASD + 90° = 180°$ [From eq. (1)]

$\Rightarrow$ $\angle ASD = 180° - 90° = 90°$

Also, $\angle RSP = \angle ASD = 90°$

[Vertically opposite angles]

Similarly, $\angle PQR = \angle QPS = \angle SRQ = 90°$

When each angle of a quadrilateral is 90°, then it is a rectangle.

So, PQRS is a rectangle. **Hence proved**

6. **Given:** In Δ ABC, AD is the median, E is the mid-point of AD and DG || BF.

To Prove: AC = 3AF

Proof: In Δ ADG,

F is the mid-point of AG.

[Converse of mid-point theorem]

$\therefore$ AF = FG ...(1)

In ΔCBF,

BF || DG, D is the mid-point of BC.

So, G is the mid-point of FC

$\therefore$ FG = GC ...(2)

From eq.(1) and (2), we get

AF = FG = GC

$\therefore$ AC = AF + FG + GC

$\Rightarrow$ AC = 3AF **Hence proved**

7. **Given:** ABCD is a parallelogram and P is the mid-point of CD.

To Prove: DA = AR and CQ = QR

Proof:

(i) In ΔDRC,

P is the mid-point of DC and AP || RC.

So, by converse of mid-point theorem,

A is the mid-point of DR.

$\Rightarrow$ DA = AR

(ii) In ΔRDC,

A is the mid-point of RD and AQ || DC

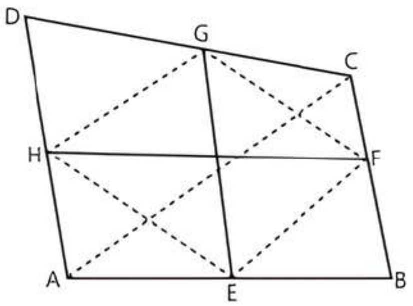
[Opposite sides of a parallelogram]

So, by converse of mid-point theorem,

Q is the mid-point of RC.

$\Rightarrow$ RQ = QC. **Hence proved**

8. **Given:** ABCD is a quadrilateral in which EG and FH are the line segments joining the mid-points of opposite sides.



To Prove: EG and FH bisect each other.

Construction: Join EF, FG, GH, HE and AC.

Proof: In ΔABC, E and F are mid-points of AB and BC respectively.

$\therefore \quad EF = \frac{1}{2}AC$ and $EF \parallel AC$ ...(1)

[By mid-point theorem]

In ΔADC, H and G are mid-points of AD and CD respectively.

$\therefore \quad HG = \frac{1}{2}AC$ and $HG \parallel AC$ ...(2)

[By mid-point theorem]

From eqs. (1) and (2), we get

$EF = HG$ and $EF \parallel HG$

TiP

In quadrilateral if one pair of opposite sides is equal and parallel, then it is a parallelogram.

$\therefore$ EFGH is a parallelogram.

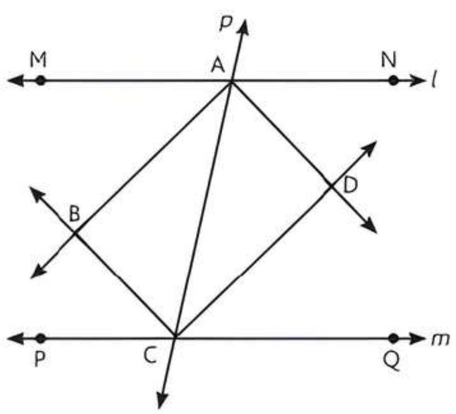
Now, EG and FH are diagonals of the parallelogram EFGH.

$\therefore$ EG and FH bisect each other.

[Diagonals of a parallelogram bisect each other]

Hence proved

9. **Given:** $l \parallel m$ and p is a transversal, AB is the bisector of $\angle MAC$, AD is the bisector of $\angle NAC$, CB is the bisector of $\angle PCA$ and CD is the bisector of $\angle QCA$.



To Prove: ABCD is a rectangle.

Proof: Since, $l \parallel m$,

$\angle MAC = \angle QCA$

[Alternate interior angles]

$\Rightarrow \quad \frac{1}{2}\angle MAC = \frac{1}{2}\angle QCA$ [Dividing by 2]

$\Rightarrow \quad \angle BAC = \angle DCA$ [$\because$ AB and CD are the bisectors of $\angle A$ and $\angle C$ respectively]

Since these are alternate interior angles, so

$AB \parallel CD$

Similarly, $AD \parallel BC$

So, quadrilateral ABCD is a parallelogram.

Now, $\angle MAC + \angle NAC = 180°$ [Linear pair]

$\Rightarrow \quad \frac{1}{2}\angle MAC + \frac{1}{2}\angle NAC = \frac{1}{2} \times 180°$

[Dividing by 2]

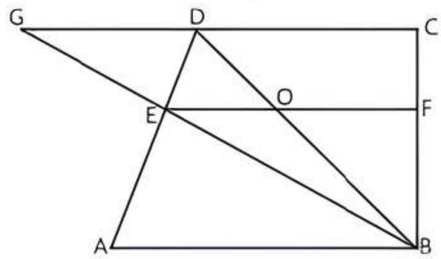
$\Rightarrow \quad \angle BAC + \angle DAC = 90°$ [$\because$ AB and AD are the bisectors of $\angle MAC$ and $\angle NAC$]

$\Rightarrow \quad \angle BAD = 90°$

A parallelogram with one angle 90° is called a rectangle.

So, ABCD is a rectangle. **Hence proved**

10. **Given:** In trapezium ABCD in which $AB \parallel CD$. Also E and F are the mid-points of AD and BC.



Construction: Join BE and produce it to meet CD produced at G, also draw BD which intersects EF at O.

To Prove: $EF \parallel AB$ and $EF = \frac{1}{2}(AB + CD)$

Proof: In ΔGCB, E and F are respectively the mid-points of GB and BC, then by mid-point theorem.

$EF \parallel GC$ and $EF = \frac{1}{2}GC$

$GC \parallel AB$ or $CD \parallel AB$ [Given]

$\therefore \quad EF \parallel AB$

In ΔADB, E is the mid-point of AD and $AB \parallel EO$.

TiP

The line drawn through the mid-point of one side of a triangle, parallel to another side bisect the third side.

By converse of mid-point theorem, O is the mid-point of BD side.

$\therefore \quad EO = \frac{1}{2}AB$ ...(1)

In ΔBDC, $OF \parallel DC$ and O and F are the mid-points of BD and BC.

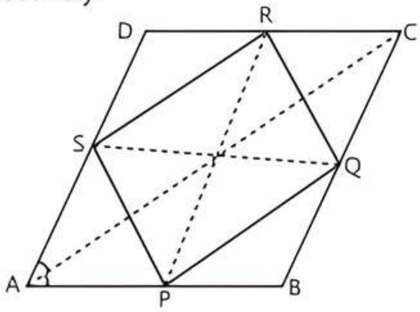
$\therefore \quad OF = \frac{1}{2}DC$ ...(2)

On adding eqs. (1) and (2), we get

$EO + OF = \frac{1}{2}AB + \frac{1}{2}DC$

$EF = \frac{1}{2}(AB + CD)$ **Hence proved**

11. Given: ABCD is a rhombus in which P, Q, R and S are mid-points of sides AB, BC, CD and DA respectively.



To Prove: PQRS is a rectangle.

Construction: Join AC, PR and SQ.

Proof: In Δ ABC,

P is the mid-point of AB and

Q is the mid-point of BC. [Given]

$\Rightarrow \quad PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ ...(1)

[By mid-point theorem]

Similarly, in Δ DAC,

$SR \parallel AC$ and $SR = \frac{1}{2}AC$ ...(2)

From eqs. (1) and (2), we get

$PQ \parallel SR$ and $PQ = SR$

$\Rightarrow$ PQRS is a parallelogram.

[One pair of opposite sides is parallel and equal]

Since, ABQS is a parallelogram.

$\Rightarrow \quad AB = SQ$ ...(3) [Opposite sides of a parallelogram]

Similarly, PBCR is a parallelogram.

$\Rightarrow \quad BC = PR$ ...(4)

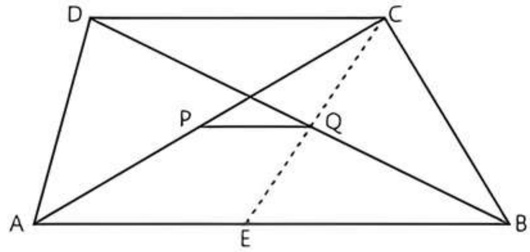
From eqs. (3) and (4)

Thus, $SQ = PR$ [$\because AB = BC$]

Since, SQ and PR are diagonals of parallelogram PQRS, which are equal.

$\Rightarrow$ PQRS is a rectangle. **Hence proved**

12. Given: ABCD is a trapezium with $AB \parallel DC$ and P, Q are the mid-points of CA and DB respectively.



To Prove: $PQ \parallel AB \parallel DC$ and

$$PQ = \frac{1}{2}(AB - DC)$$

Construction: Join CQ and produce it to meet AB at E.

Proof: In ΔCDQ and ΔEBQ,

$DQ = BQ$ [Given]

$\angle DCQ = \angle BEQ$

[Alternate interior angles since, $\angle DCQ = \angle DCE$ and $\angle BEQ = \angle BEC$]

$\angle CDQ = \angle EBQ$ [Alternate interior angles]

$\therefore \quad \Delta CDQ \cong \Delta EBQ$ [By AAS congruence rule]

Thus, $CQ = QE$ [By CPCT]

$DC = EB$ [By CPCT]

Now, in ΔCEA,

Q is the mid-point of CE [Proved above]

P is the mid-point of CA. [Given]

So, by mid-point theorem,

$PQ \parallel AE$ and $PQ = \frac{1}{2}AE$

So, $PQ \parallel AB \parallel DC$ [$\because PQ \parallel AB$ and $AB \parallel DC$]

and $PQ = \frac{1}{2}AE = \frac{1}{2}(AB - EB) = \frac{1}{2}(AB - DC)$

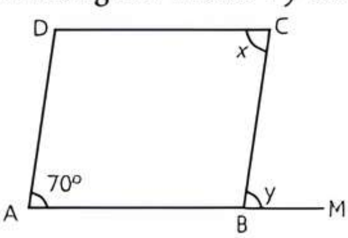
[$\because EB = DC$ proved above]

Hence proved

Chapter Test

Multiple Choice Questions

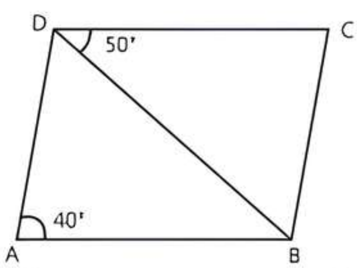
Q 1. If ABCD is a parallelogram in which $\angle DAB = 70°$ and AB is produced to point M as shown in the figure. Then $x + y$ is:



a. 70° b. 140° c. 120° d. 130°

Q 2. In the given figure, ABCD is a parallelogram in which $\angle BDC = 50°$ and $\angle BAD = 40°$. Then $\angle CBD$ is equal to:

a. 80° b. 70° c. 90° d. 50°



Assertion and Reason Type Questions

Directions (Q.Nos. 3-4) *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

c. Assertion (A) is true but Reason (R) is false.

d. Assertion (A) is false but Reason (R) is true.

Q 3. Assertion (A): The opposite angles of a parallelogram are $(2x - 3)°$ and $(54 - x)°$. The measure of one of the angle is $52°$.

Reason (R): Opposite angles of a parallelogram are equal.

Q 4. Assertion (A): Diagonals AC and BD of a parallelogram in ABCD intersect each other at point O. If $\angle DAC = 40°$ and $\angle AOB = 60°$, then $\angle DBC = 20°$.

Reason (R): The adjacent angles of a parallelogram is supplementary.

Fill in the Blanks

Q 5. The diagonals of a square bisect each other at ....

Q 6. The line segment joining the mid-points of the two sides of the triangle is .... to the third side.

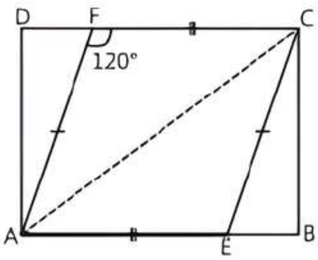
True/False

Q 7. In a parallelogram, diagonals, bisect each other.

Q 8. The sum of the adjacent angles of a parallelogram is 90°.

Case Study Based Question

Q 9. An organisation was donated with some land for charity, which is in the shape of parallelogram AECF. The organisation was added some piece of land and converted it into a rectangular plot by adding triangular land ΔADF and ΔBCF.



On the basis of the above information, solve the following questions:

(i) Write the measure of $\angle AFD$.

(ii) What is the measure of $\angle AEC$?

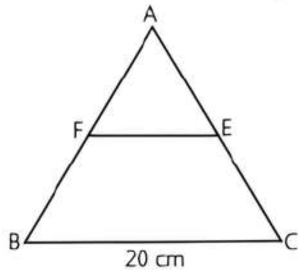
OR

In which criteria, ΔADF and ΔCBF is congruent.

(iii) What is the measure of $\angle FAE$?

Very Short Answer Type Questions

Q 10. In triangle ABC, if F and E are the mid points of AB and AC, then find the length of FE.



Q 11. Two consecutive angles of a parallelogram are in the ratio 2 : 3, then what will be the smaller angle?

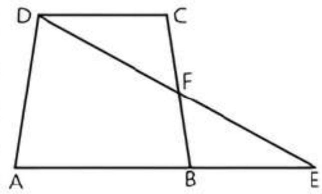
Short Answer Type-I Questions

Q 12. A diagonal of a rectangle is inclined to one side of the rectangle at 45°. Find the angle between the diagonals.

Q 13. The angle between the two altitudes of a parallelogram through the vertex of an obtuse angle is 60°. Find the angles of a parallelogram.

Short Answer Type-II Questions

Q 14. In the figure, ABCD is a trapezium with AB || DC. F is the mid-point of BC. DF and AB are produced to meet at E. Show that F is also the mid-point of DE.



Q 15. P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD in which AC = BD. Prove that PQRS is a rhombus.

Long Answer Type Question

Q 16. In a parallelogram ABCD, E and F are the mid points of sides AB and CD, respectively. Show that the line segments AF and EC trisect the diagonal BD.